Entropies of Rotating Charged Black Holes from Conformal Field Theory at Killing Horizons

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Abstract

The covariant phase technique is used to compute the constraint algebra of the stationary axisymmetric charged black hole. A standard Virasoro subalgebra with corresponding central charge is constructed at a Killing horizon with Carlip's boundary conditions. For the Kerr-Newman black hole and the Kerr-Newman-AdS black hole, the density of states determined by conformal fields theory methods yields the statistical entropy which agrees with the Bekenstein-Hawking entropy.

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I. INTRODUCTION

The statistical mechanical description of the Bekenstein-Hawking black hole entropy [1]-[3] in terms of microscope states is an outstanding open question and much effort has been concentrated on the problem for some years [4]- [22]. Success seems to come with the paper of Strominger and Vafa [23] which was followed by a host of others. It is well known since the work of Brown and Henneaux [24] that a asymptotic symmetry group of AdS_3 is generated by a Virasoro algebra, and that therefore any consistent quantum theory of gravity on AdS_3 is conformal field theory. Using the result Strominger [25] calculated the entropy of black holes whose near-horizon geometry is locally AdS_3 from the asymptotic growth of states. Precise numerical agreement with the Bekenstein-Hawking area formula for the entropy was found. In light of the work, one could statistically reinterpret the black hole entropy by establishing a relation conformal field theory on the boundary of related anti-de Sitter space. In order to overcome the limitations of Strominger's method, such as the approach can only be used for 2+1 dimensional spacetime and it is based on an algebra of transformations at infinity, Carlip [26] generalized Brown-Henneaux-Strominger's approach by looking at the symmetries of the event horizon of an (n+1)-dimensional Schwarzschild-like black hole. This construction is valid for black hole in any dimension. In Ref. [27] Carlip re-derived the central extension of the constraint algebra of general relativity by using manifestly covariant phase space methods [28]- [31] and a boundary which is a surface that look like a (local) Killing horizon. A natural set of boundary conditions leads to a Virasoro subalgebra with a calculable central charge. Then, by means of conformal field theory method, Carlip [27] studied the statistical entropies of the Rindler space, static de Sitter space, Taub-NUT and Taub-Bolt spaces, and 2-dimensional dilaton gravity. However, at the moment the question whether or not the covariant phase space approach can be used for the stationary axisymmetric charged black holes which are described by solutions of the Einstein-Maxwell equations, such as the Kerr-Newman black hole and the Kerr-Newman-AdS black hole, still remains open. The aim of this paper is to settle the question.

The paper is organized as follows: In Sec. II, by using the covariant phase techniques we extend Carlip's investigation [27] for vacuum case $\mathbf{L}_{a_1a_2\cdots a_n} = \frac{1}{16\pi G}\epsilon_{a_1a_2\cdots a_n}R$ to a case including a cosmological term and electromagnetic fields, i.e., the Lagrangian n-form is described by $\mathbf{L}_{a_1a_2\cdots a_n} = \frac{1}{16\pi}\epsilon_{a_1a_2\cdots a_n}\left[\frac{1}{G}(R-2\Lambda)+F^{ab}F_{ab}\right]$. In Sec. III, the standard Virasoro subalgebras with corresponding central charges are constructed for the Kerr-Newman black hole and the Kerr-Newman-AdS black hole. The statistical entropies for these objects are then calculated by using Cardy formula. Some discussions and summaries are presented in the last section.

II. ALGEBRA OF DIFFEOMORPHISM

Lee, Wald, and Iyer [28] [29] [30] [31] showed that the variation of the Lagrangian defines the equation of motion n-form $\mathbf{E}$ and the symplectic potential (n-1)-form $\mathbf{\Theta}$ via the equation $\delta\mathbf{L} = \mathbf{E}\delta\phi + d\mathbf{\Theta}$, where $\mathbf{L}$ is an n-form, $\mathbf{E}\delta\phi = \mathbf{E}_g^{ab}\delta g_{ab} + \mathbf{E}_\psi\delta\psi$, $\phi = (g_{ab},\psi)$ denotes an arbitrary collection of dynamical fields, and the equations of motion are taken to be $\mathbf{E}_g^{ab} = 0$

and $\mathbf{E}_\psi = 0$. Let ξ^a be any smooth vector fields on the spacetime manifold $\mathbf{M}$, i. e., ξ^a is the infinitesimal generator of a diffeomorphism, we can define a Noether current (n-1)-form as [30] [31]

$$\mathbf{J}[\xi] = \mathbf{\Theta}[\phi, \mathcal{L}_\xi \phi] - \xi \cdot \mathbf{L}, \tag{2.1}$$

here and hereafter the "central dot" denotes the contraction of the vector field ξ^a into the first index of the differential form. By using the equations of motion a standard calculation [28] shows that $\mathbf{J}$ is closed for all ξ^a, i.e., $d\mathbf{J} = 0$. Then we have [30]

$$\mathbf{J} = d\mathbf{Q}, \tag{2.2}$$

where $\mathbf{Q}$ is a Noether charge (n-2)-form. From the variation of Noether current (n-1)-form, we know that the symplectic current (n-1)-form $\omega[\phi, \delta_1\phi, \delta_2\phi] = \delta_2\mathbf{\Theta}[\phi, \delta_1\phi] - \delta_1\mathbf{\Theta}[\phi, \delta_2\phi]$ can be expressed as [28]

$$\omega[\phi, \delta\phi, \mathcal{L}_\xi\phi] = \delta\mathbf{J}[\xi] - d(\xi \cdot \mathbf{\Theta}[\phi, \delta\phi]), \tag{2.3}$$

and Hamilton's equation of motion is given by

$$\delta H[\xi] = \int_C \omega[\phi, \delta\phi, \mathcal{L}_\xi\phi] = \int_C [\delta\mathbf{J}[\xi] - d(\xi \cdot \mathbf{\Theta}[\phi, \delta\phi])]. \tag{2.4}$$

By using Eq. (2.2) and Carlip's boundary conditions listed in Appendix A and defining a (n-1)-form $\mathbf{B}$ as

$$\delta \int_{\partial C} \xi \cdot \mathbf{B}[\phi] = \int_{\partial C} \xi \cdot \mathbf{\Theta}[\phi.\delta\phi], \tag{2.5}$$

the Hamiltonian can be expressed as [27]

$$H[\xi] = \int_{\partial C} (\mathbf{Q}[\xi] - \xi \cdot \mathbf{B}[\phi]). \tag{2.6}$$

It is well-known that the Poisson bracket forms a standard "surface deformation algebra" [24] [27]

$$\{H[\xi_1], H[\xi_2]\} = H[\{\xi_1, \xi_2\}] + K[\xi_1, \xi_2], \tag{2.7}$$

where the central term $K[\xi_1, \xi_2]$ depends on the dynamical fields only through their boundary values.

In this paper, we focus our attention to stationary axisymmetric charged black holes. So we take the Lagrangian n-form as

$$\mathbf{L}_{a_1 a_2 \cdots a_n} = \frac{1}{16\pi} \epsilon_{a_1 a_2 \cdots a_n} \left[\frac{1}{G}(R - 2\Lambda) + F^{ab}F_{ab} \right], \tag{2.8}$$

where $\epsilon_{a_1 a_2 \cdots a_n}$ is a volume element (a continuous non-vanishing n-form), Λ is the cosmological constant, and F_{ab} is the electromagnetic field strength tensor. By using the infinitesimal generator of a diffeomorphism, ξ, we know that the symplectic potential (n-1)-form is given by

$$\mathbf{\Theta}_{a_1a_2\cdots a_{n-1}}[g,\mathcal{L}_\xi g]=\frac{1}{4\pi}\epsilon_{ca_1a_2\cdots a_{n-1}}\left\{\frac{1}{2G}(\nabla_e\nabla^{[e}\xi^{c]}+R^c_e\xi^e)+F^{dc}\left[F_{ed}\xi^e+(\xi^eA_e)_{;d}\right]\right\}. \quad (2.9)$$

Eqs. (2.1) and (2.9) yields

$$\begin{aligned}\mathbf{J}_{a_1a_2\cdots a_{n-1}}&=\frac{1}{8\pi G}\epsilon_{ca_1a_2\cdots a_{n-1}}[\nabla_e\nabla^{[e}\xi^{c]}+(R^c_e-\frac{1}{2}\delta^c_eR+\delta^c_e\Lambda)\xi^e]\\&-\frac{1}{4\pi}\epsilon_{ca_1a_2\ldots a_{n-1}}\left[\frac{1}{4}F^{bd}F_{bd}\delta^c_e-F^{cd}F_{ed}\right]\xi^e+\frac{1}{4\pi}\epsilon_{ca_1a_2\ldots a_{n-1}}F^{ec}(\xi^dA_d)_{;e}\\&=\frac{1}{4\pi}\epsilon_{ca_1a_2\cdots a_{n-1}}\left[\frac{1}{2G}\nabla_e\nabla^{[e}\xi^{c]}+\nabla_e(\nabla^{[e}A^{c]}A_d\xi^d)\right], \quad (2.10)\end{aligned}$$

in above calculation, we used the Einstein-Maxwell field equations in which the energy-momentum tensors is given by $\frac{1}{4\pi}\left[\frac{1}{4}F^{bc}F_{bc}\delta^d_e-F^{dc}F_{ec}\right]$.

From Eqs. (2.2) and (2.10) we have

$$\mathbf{Q}_{a_1a_2\cdots a_{n-2}}=-\frac{1}{4\pi}\epsilon_{bca_1\ldots a_{n-2}}\left[\frac{1}{4G}\nabla^b\xi^c+(\nabla^bA^c)A_e\xi^e\right]. \quad (2.11)$$

For a stationary axisymmetric charged black hole (such as the Kerr-Newman black hole and the Kerr-Newman AdS/dS black hole), the electromagnetic potential A_a, the electromagnetic field tensors F^{03}, and the Killing vector can be expressed respectively as

$$A_a=(A_0,\ 0,\ 0,\ A_3)$$

$$F^{03}=-F^{30}=0. \quad (2.12)$$

$$\chi^a_H=\chi^{(t)}_H+\chi^{(\varphi)}_H=(1,\ 0,\ 0,\ \Omega_H), \quad (2.13)$$

where the vector $\chi^{(t)}_H$ correspond to time translation invariance, $\chi^{(\varphi)}_H$ to rotational symmetry, and $\Omega_H=-(g_{t\varphi}/g_{\varphi\varphi})_H$ is the angular velocity of the black hole.

Using Eqs. (2.12), (2.13), (A5), and (A9) it is easy to show that

$$\frac{1}{4\pi}\epsilon_{bca_1\ldots a_{n-2}}(\nabla^bA^c)A_e\xi^e\to 0. \qquad \text{at the horizon} \quad (2.14)$$

Then, Eq. (2.11) is reduced to

$$Q_{a_1a_2\ldots a_{n-2}}=-\frac{1}{16\pi G}\epsilon_{bca_1a_2\ldots a_{n-2}}\nabla^b\xi^c. \quad (2.15)$$

Denoting by δ_ξ the variation corresponding to diffeomorphism generated by ξ, for the Noether current $\mathbf{J}[\xi]$ we have

$$\delta_{\xi_2}\mathbf{J}[\xi_1]=\xi_2d\mathbf{J}[\xi_1]+d(\xi_2\cdot\mathbf{J}[\xi_1])=d[\xi_2(\mathbf{\Theta}[\phi,\mathcal{L}_{\xi_1}\phi]-\xi_1\cdot\mathbf{L})]. \quad (2.16)$$

Substituting Eq. (2.16) into Eq. (2.4) and using Eq. (2.9) we get

$$\begin{aligned}\delta_{\xi_2}H[\xi_1]&=\int_C(\delta_{\xi_2}\mathbf{J}[\xi_1]-d(\xi_1\mathbf{\Theta}[\phi,\delta_{\xi_2}\phi]))\\&=\int_{\partial C}(\xi_2\mathbf{\Theta}[\phi,\mathcal{L}_{\xi_1}\phi]-\xi_1\mathbf{\Theta}[\phi,\mathcal{L}_{\xi_2}\phi]-\xi_2\xi_1\mathbf{L})\\&=\frac{1}{16\pi G}\int_{\partial C}\epsilon_{bca_1\ldots a_{n-2}}\left[\xi^b_2\nabla_d(\nabla^d\xi^c_1-\nabla^c\xi^d_1)-\xi^b_1\nabla_d(\nabla^d\xi^c_2-\nabla^c\xi^d_2)\right]\\&+\frac{1}{8\pi}\int_{\partial C}\epsilon_{bca_1\ldots a_{n-2}}\left\{\xi^b_2F^{dc}\left[F_{ed}\xi^e_1+(\xi^e_1A_e)_{;d}\right]-\xi^b_1F^{dc}\left[F_{ed}\xi^e_2+(\xi^e_2A_e)_{;d}\right]\right\}\\&-\frac{1}{16\pi G}\int_{\partial C}\epsilon_{bca_1\ldots a_{n-2}}\left[2R^c_d(\xi^b_1\xi^d_2-\xi^b_2\xi^d_1)+\xi^b_2\xi^c_1\mathbf{L}\right]. \quad (2.17)\end{aligned}$$

At the horizon, by using Eqs. (2.12), (2.13) and (A4)- (A9) we know

$$\begin{aligned}
&\int_{\partial C} \epsilon_{bca_1\ldots a_{n-2}} \xi_2^b \xi_1^c \mathbf{L} \\
&= \int_{\partial C} \hat{\epsilon}_{a_1\ldots a_{n-2}} \mathbf{L} \left[\frac{|\chi|}{\rho} \mathcal{T}_2 \rho_c + \left(\frac{\rho}{|\chi|} + t\cdot\rho \right) \mathcal{R}_2 \chi_c \right] (\mathcal{T}_1 \chi^c + \mathcal{R}_1 \rho^c) \\
&= \int_{\partial C} \hat{\epsilon}_{a_1\ldots a_{n-2}} \mathbf{L} \left[\frac{|\chi|}{\rho} \mathcal{T}_2 \mathcal{R}_1 \rho^2 + \left(\frac{\rho}{|\chi|} + t\cdot\rho \right) \mathcal{R}_2 \mathcal{T}_1 \chi^2 \right] \\
&= 0,
\end{aligned} \tag{2.18}$$

$$\begin{aligned}
&\int_{\partial C} \epsilon_{bca_1\ldots a_{n-2}} 2R^c_d (\xi_1^b \xi_2^d - \xi_2^b \xi_1^d) \\
&= \int_{\partial C} \hat{\epsilon}_{a_1\ldots a_{n-2}} R^c_d \left(\frac{1}{\kappa} \frac{\chi^2}{\rho^2} \right) \left[\frac{|\chi|}{\rho} \rho_c \rho^d - \left(\frac{\rho}{|\chi|} + t\cdot\rho \right) \chi_c \chi^d \right] (\mathcal{T}_1 D \mathcal{T}_2 - \mathcal{T}_2 D \mathcal{T}_1) \\
&= 0,
\end{aligned} \tag{2.19}$$

and

$$\begin{aligned}
&\xi^b \epsilon_{bca_1a_2\ldots a_{n-2}} F^{dc} \left[F_{ed} \xi^e + (\xi^e A_e)_{;d} \right] \\
&= \xi^b \epsilon_{bca_1a_2\ldots a_{n-2}} F^{dc} \delta_\xi A_d \\
&= \hat{\epsilon}_{a_1a_2\ldots a_{n-2}} \left[\frac{|\chi|}{\rho} \mathcal{T} \rho_c + \left(\frac{\rho}{|\chi|} + t\cdot\rho \right) \mathcal{R} \chi_c \right] F^{dc} \delta_\xi A_d \\
&= 0.
\end{aligned} \tag{2.20}$$

Therefore, Eq. (2.17) can be rewritten as

$$\delta_{\xi_2} H[\xi_1] = \frac{1}{16\pi G} \int_{\partial C} \epsilon_{bca_1\ldots a_{n-2}} \left[\xi_2^b \nabla_d (\nabla^d \xi_1^c - \nabla^c \xi_1^d) - \xi_1^b \nabla_d (\nabla^d \xi_2^c - \nabla^c \xi_2^d) \right]. \tag{2.21}$$

Since the "bulk" part of the generator $H[\xi_1]$ on the left side vanishes on shell, we can interpret the left side of Eq. (2.17) the variation of the boundary term J, i.e., $\delta_{\xi_2} J[\xi_1]$. On the other hand, the change in $J[\xi_1]$ under a surface deformation generated by $J[\xi_2]$ can be precisely described by Dirac bracket $\{J[\xi_1], j[\xi_2]\}^*$ [27], that is,

$$\delta_{\xi_2} J[\xi_1] = \{J[\xi_1], J[\xi_2]\}^* = \frac{1}{16\pi G} \int_{\partial C} \epsilon_{bca_1\ldots a_{n-2}} \left[\xi_2^b \nabla_d (\nabla^d \xi_1^c - \nabla^c \xi_1^d) - \xi_1^b \nabla_d (\nabla^d \xi_2^c - \nabla^c \xi_2^d) \right]. \tag{2.22}$$

Above discussions and Eq. (2.7) show the following relation on shell

$$\{J[\xi_1], J[\xi_2]\}^* = J[\{\xi_1, \xi_2\}] + K[\xi_1, \xi_2], \tag{2.23}$$

Substituting Eqs. (A5), (A6), and (A9) into Eq. (2.22) we find that

$$\{J[\xi_1], J[\xi_2]\}^* = -\frac{1}{16\pi G} \int_{\partial C} \hat{\epsilon}_{a_1\cdots a_{n-2}} \left[\frac{1}{\kappa} (\mathcal{T}_1 D^3 \mathcal{T}_2 - \mathcal{T}_2 D^3 \mathcal{T}_1) - 2\kappa (\mathcal{T}_1 D \mathcal{T}_2 - \mathcal{T}_2 D \mathcal{T}_1) \right]. \tag{2.24}$$

It is also easy to show that

$$\{\xi_1,\xi_2\}^a=(\mathcal{T}_1D\mathcal{T}_2-\mathcal{T}_2D\mathcal{T}_1)\chi^a+\frac{1}{\kappa}\frac{\chi^2}{\rho^2}D(\mathcal{T}_1D\mathcal{T}_2-\mathcal{T}_2D\mathcal{T}_1)\rho^a. \tag{2.25}$$

On the other hand, the integrand of the right hand of Eq. (2.5) can be expressed as

$$\xi^b\mathbf{\Theta}_{ba_1\ldots a_{n-2}}=\frac{1}{4\pi}\xi^b\epsilon_{bca_1\cdots a_{n-2}}\left\{\frac{1}{2G}(\nabla_e\nabla^{[e}\xi^{c]}+R^c_e\xi^e)+F^{dc}\left[F_{ed}\xi^e+(\xi^eA_e)_{;d}\right]\right\}. \tag{2.26}$$

The first two terms in the right hand of Eq. (2.26) can be treated as Carlip did in Ref. [27]. And by using Eqs. (2.20) we know that the last two terms in Eq. (2.26) gives no contribution to $K[\xi_1,\xi_2]$. Making use of Eqs. (2.2), (2.15), (A5), (A6), and (A9) and replacing ξ^a in J by $\{\xi_1,\xi_2\}^a$, we have

$$J[\{\xi_1,\xi_2\}]=\frac{1}{16\pi G}\int_{\partial C}\hat{\epsilon}_{a_1a_2\cdots a_{n-2}}\left[2\kappa(\mathcal{T}_1D\mathcal{T}_2-\mathcal{T}_2D\mathcal{T}_1)-\frac{1}{\kappa}D(\mathcal{T}_1D^2\mathcal{T}_2-\mathcal{T}_2D^2\mathcal{T}_1)\right]. \tag{2.27}$$

The central term can then be obtained from Eqs.(2.23), (2.24), and (2.27), which is explicitly given by

$$K[\xi_1,\xi_2]=\frac{1}{16\pi G}\int_{\partial C}\hat{\epsilon}_{a_1a_2\cdots a_{n-2}}\frac{1}{\kappa}(D\mathcal{T}_1D^2\mathcal{T}_2-D\mathcal{T}_2D^2\mathcal{T}_1). \tag{2.28}$$

III. ENTROPY OF SOME ROTATING CHARGED BLACK HOLES

In this section, lets us study statistical-mechanical entropies of the stationary axisymmetric black holes by using the constraint algebra constructed in the preceding section and conformal field theory methods.

A. Entropy of the Kerr-Newman black hole

In Boyer-Lindquist coordinates, the metric of the Kerr-Newman black hole takes the form [32] [33]

$$ds^2=-\frac{\Delta}{\varrho^2}\left[dt-a\sin^2\theta d\varphi\right]^2+\frac{\varrho^2}{\Delta}dr^2+\varrho^2d\theta^2+\frac{\sin^2\theta}{\varrho^2}\left[adt-(r^2+a^2)d\varphi\right]^2, \tag{3.1}$$

with

$$\begin{aligned}\varrho^2&=r^2+a^2\cos^2\theta,\\ \Delta&=(r-r_+)(r-r_-),\end{aligned} \tag{3.2}$$

where $r_+=r_H=M+\sqrt{M^2-Q^2-a^2}$, $r_-=M-\sqrt{M^2-Q^2-a^2}$, the parameter a is related to the angular momentum, and M and Q represent the mass and electric charge of the black hole, respectively. The metric (3.1) is a solution of the Einstein-Maxwell field equations with an electromagnetic vector potential

$$\mathbf{A} = -\frac{Qr}{\varrho^2}(dt - asin^2\theta d\varphi), \tag{3.3}$$

and associated field strength tensor

$$\begin{aligned} \mathbf{F} = & -\frac{Q}{\varrho^4}(r^2 - a^2\cos^2\theta)e^0 \wedge e^1 \\ & + \frac{Q}{\varrho^4}(r^2 - a^2\cos^2\theta)e^2 \wedge e^3. \end{aligned} \tag{3.4}$$

The Killing vector can be expressed as

$$\chi_H^a = (1,\ 0,\ 0,\ \Omega_H), \tag{3.5}$$

where $\Omega_H = -\left(\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right)_H = \frac{a}{r^2+a^2}$ is the angular velocity of the black hole. A one-parameter group of diffeomorphism satisfying Eqs. (A8) and (2.25) can be taken as

$$\mathcal{T}_n = \frac{1}{\kappa}exp\left[in(\kappa t + C_\alpha(\varphi - \Omega_H t))\right], \tag{3.6}$$

where C_α is a arbitrary constant. Substituting Eq. (3.6) into central term (2.28) and using condition (A8) we obtain

$$K[\mathcal{T}_m, \mathcal{T}_n] = -\frac{iA_H}{8\pi G}m^3\delta_{m+n,0}, \tag{3.7}$$

where $A_H = \int_{\partial C}\hat{\epsilon}_{a_1a_2\ldots a_{n-2}} = 4\pi(r_+^2 + a^2)$ is the area of the event horizon. Thus, Eq. (2.23) takes standard form of a Virasoro algebra

$$i\{J[\mathcal{T}_m], J[\mathcal{T}_n]\} = (m-n)J[\mathcal{T}_{m+n}] + \frac{c}{12}m^3\delta_{m+n,0}, \tag{3.8}$$

with central charge $\frac{c}{12} = \frac{A_H}{8\pi G}$. The boundary term $J[\mathcal{T}_0]$ can easily be obtained by using Eqs (2.2), (2.11), and (3.6), which is given by $J[\mathcal{T}_0] = \frac{A_H}{8\pi G}$. The number of states with a given eigenvalue $\triangle$ of $J[\mathcal{T}_0]$ grows asymptotically for large $\triangle$ as

$$\rho(\triangle) \sim exp\left\{2\pi\sqrt{\frac{c}{6}\left(\triangle - \frac{c}{24}\right)}\right\} = exp\left[\frac{A_H}{4G}\right], \tag{3.9}$$

and the statistical entropy of the Kerr-Newman black hole is

$$\log\rho(\triangle) \sim \frac{\mathrm{A_H}}{\mathrm{4G}}, \tag{3.10}$$

which coincides with the standard Bekenstein-Hawking entropy.

B. Entropy of the Kerr-Newman-AdS black hole

Carter [34] constructed the Kerr-Newman-AdS black hole in four dimensions many years ago, which can be explicitly given by

$$ds^2 = -\frac{\Delta_r}{\varrho^2}\left[dt - \frac{a}{\Xi}\sin^2\theta d\varphi\right]^2 + \frac{\varrho^2}{\Delta_r}dr^2 + \frac{\varrho^2}{\Delta_\theta}d\theta^2 + \frac{\sin^2\theta\Delta_\theta}{\varrho^2}\left[adt - \frac{(r^2+a^2)}{\Xi}d\varphi\right]^2, \quad (3.11)$$

with

$$\begin{aligned}\varrho^2 &= r^2 + a^2\cos^2\theta, \\ \Delta_r &= (r^2+a^2)(1+l^2r^2) - 2Mr + q^2 + p^2, \\ \Delta_\theta &= 1 - l^2a^2\cos^2\theta, \\ \Xi &= 1 - l^2a^2,\end{aligned}$$

where the parameter M is related to the mass, a to the angular momentum, q is proportional to the electric charge, p is proportional to the magnetic charge, and $l^2 = -\Lambda/3$ (where Λ is the (negative) cosmological constant). The event horizon is located at $r = r_+$, the largest root of the polynomial Δ_r. The metric (3.11) is a solution of the Einstein-Maxwell field equations with an electromagnetic vector potential is given by

$$\mathbf{A} = -\frac{qr}{\varrho^2\Xi}(dt - asin^2\theta d\varphi) - \frac{p\cos\theta}{\varrho^2\Xi}[adt - (r^2+a^2)d\varphi], \quad (3.12)$$

and the associated field strength tensor is

$$\begin{aligned}\mathbf{F} = &-\frac{1}{\varrho^4}[q(r^2 - a^2\cos^2\theta) + 2pra\cos\theta]e^0\wedge e^1 \\ &+\frac{1}{\varrho^4}[q(r^2 - a^2\cos^2\theta) - 2pra\cos\theta]e^2\wedge e^3.\end{aligned} \quad (3.13)$$

The Killing vector is

$$\chi_H^a = \partial_t + \Omega_H\partial_\varphi, \quad (3.14)$$

where $\Omega_H = -\left(\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right)_H = \frac{\Xi a}{r_+^2 + a^2}$ is the angular velocity of the black hole. The analysis of the preceding subsection goes through with virtually no changes, yields a statistical entropy

$$S = \frac{A_H}{4G} = \frac{\pi}{G}\frac{r_+^2 + a^2}{\Xi}, \quad (3.15)$$

which also coincides with its Bekenstein-Hawking entropy.

IV. SUMMARY AND DISCUSSION

By using the covariant phase techniques we extend Carlip's investigation in Ref. [27] to a case containing a cosmological term and a electromagnetic field. If the event horizon is treated as a boundary with Carlip's constraint conditions [27], the central extension of the constraint algebra is worked out, and a standard Virasoro subalgebra with a corresponding central charge is constructed for the stationary axisymmetric charged black hole. The statistical entropies of the Kerr-Newman black hole and the Kerr-Newman-AdS black hole are then obtained by using Cardy formula and the results agree with their Bekenstein-Hawking entropies.

Since the static charged black holes, such as the Reissner-Nordström black hole and Reissner-Nordström-AdS black hole are special case of the metrics (3.1) and (3.11), respectively, the above results are also valid for the static charged black holes.

The results obtained in this paper support Carlip's supposition: regardless of the details of a quantum theory of gravity, symmetries inherited from the classical theory may be sufficient to determine the asymptotic behavior of the density of states.

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APPENDIX A: BOUNDARY CONDITIONS

In this section, we list the Carlip's boundary conditions [27] for convenience.

As Carlip did in Ref. [27] we define a "stretched horizon"

$$\chi^2 = \epsilon. \tag{A1}$$

where $\chi^2 = g_{ab}\chi^a\chi^b$, χ^a is a Killing vector. The result of the computation will be evaluated at the event horizon of the black hole by taking ϵ to zero. Near the stretched horizon, one can introduce a vector orthogonal to the orbit of χ^a by

$$\nabla_a \chi^2 = -2\kappa\rho_a, \tag{A2}$$

where κ is the surface gravity. Vector ρ^a satisfies conditions

$$\begin{aligned} \chi^a\rho_a = -\frac{1}{\kappa}\chi^a\chi^b\nabla_a\chi_b = 0, &\qquad \text{everywhere} \\ \rho^a \to \chi^a, &\qquad \text{at the horizon} \end{aligned} \tag{A3}$$

To preserve the "asymptotic" structure at horizon, we impose Carlip's boundary conditions [27]

$$\delta\chi^2 = 0, \qquad \chi^a t^b \delta g_{ab} = 0, \qquad \delta\rho_a = -\frac{1}{2\kappa}\nabla_a(\delta\chi^2) = 0, \qquad at \quad \chi^2 = 0, \tag{A4}$$

where t^a is a any unit spacelike vector tangent to boundary $\partial\mathbf{M}$ of the spacetime $\mathbf{M}$. And the surface deformation vector is suggested as the following form

$$\xi^a = \mathcal{R}\rho^a + \mathcal{T}\chi^a, \tag{A5}$$

where functions $\mathcal{R}$ and $\mathcal{T}$ satisfy [27]

$$\begin{aligned} \mathcal{R} = \frac{1}{\kappa}\frac{\chi^2}{\rho^2}\chi^a\nabla_a\mathcal{T}, &\qquad \text{everywhere} \\ \rho^a\nabla_a\mathcal{T} = 0, &\qquad \text{at the horizon.} \end{aligned} \tag{A6}$$

Fixing the average value of $\tilde{\kappa}$ ($\tilde{\kappa} = -\frac{a^2}{\chi^2}$, $a^a = \chi^b\nabla_b\chi^a$ is the acceleration of an orbit of χ^a) over a cross section of the horizon [27]

$$\delta \int_{\partial C} \hat{\epsilon}\left(\tilde{\kappa} - \frac{\rho}{|\chi|}\kappa\right) = 0, \tag{A7}$$

where κ is the surface gravity and $\hat{\epsilon}$ is the induced volume measure on $\mathcal{H}$ ($\mathcal{H}$ denote the (n-2)-dimensional intersection of the Cauchy surface C with the Killing horizon $\chi^2 = 0$). The technical role of the condition (A7) is to guarantee the existence of generators $H[\xi]$. For a one-parameter group of diffeomorphism such that $D\mathcal{T}_\alpha = \lambda_\alpha\mathcal{T}_\alpha$, ($D \equiv \chi^a\partial_a$), condition (A7) in turn implies an orthogonality relation [27]

$$\int_{\partial C} \hat{\epsilon}\ \mathcal{T}_\alpha\mathcal{T}_\beta \sim \delta_{\alpha+\beta}. \tag{A8}$$

By using the other future-directed null normal vector $N^a = k^a - \alpha\chi^a - t^a$, with $k^a = -\frac{1}{\chi^2}\left(\chi^a - \frac{|\chi|}{\rho}\rho^a\right)$ and a normalization $N_a\chi^a = -1$, the volume element can be expressed as

$$\epsilon_{bca_1\cdots a_{n-2}} = \hat{\epsilon}_{a_1\cdots a_{n-2}}(\chi_b N_c - \chi_c N_b) + \cdots\cdots, \tag{A9}$$

the omitted terms do not contribute to the integral.

REFERENCES

[1] J.D. Bekenstein, Nuovo Cimento Lett. **4**, (1972) 737; Phys. Rev. D., **7**, (1973) 2333.

[2] J Bekenstein, Phys. Rev. D **9**, (1974) 3292; R. Kallosh, T. Ortin, and A. Peet, Phys. Rev. D **47**, (1993) 5400.

[3] S. W. Hawking, Nature (London). **248**, (1974) 30; Commun. Math. Phys., **43**, (1975) 199.

[4] G. 't. Hooft, Nucl. Phys. B, **256**, (1985) 727.

[5] J.G.Demers, R. Lafrance and R.C. Myers, Phys. Rev. D **52**, (1995) 2245.

[6] A. Ghosh and P. Mitra, Phys. Rev. Lett. **73**, (1994) 2521.

[7] V. P. Frolov and D. V. Fursaev, Class. Quantum Grav. **15**, (1998) 2041.

[8] M.H. Lee and J. K. Kim, Phys. Lett. A **212**, (1996) 323.

[9] R. B. Mann, L. Tarasov and A. Zelnikov, Class. Quantum Grav. **9**, (1992) 1487.

[10] A. Romeo, Class. Quantum Grav. **13**, (1996) 2797.

[11] S. P. Kim et al Phys. Rev. D **55**, (1997) 2159.

[12] M.H. Lee and J. K. Kim, Phys. Rev. D **54**, (1996) 3904.

[13] M.H. Lee, H. C. Kim and J. K. Kim, Phys. Lett. B **388**, (1996) 487.

[14] S. W. Kim, W. T. Kim, Y. J. Park and H. Shin, Phys. Lett. B **392**, (1997) 311.

[15] J. Ho, W. T. Kim and Y. J. Park, "Entropy in the Kerr-Newman black hole". gr-qc/9704032, 1997.

[16] F. Belgiorno and S. Liberati, Phys. Rev. D **53**, (1996) 3172.

[17] Jiliang Jing, Chin. Phys. Lett. **14**, (1997) 495.

[18] Jiliang Jing, Inter. J. Theor. Phys. **37** (1998) 1441.

[19] Jiliang Jing and Mu-Lin Yan, gr-qc/9904001; Phys. Rev. D **60**, 084015 (1999).

[20] Jiliang Jing and Mu-Lin Yan, gr-qc/9907011; Phys. Rev. D **61**, 044016 (2000).

[21] I-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee. On the energy of a charged dilaton black hole, NCKU-HEP/96-03.

[22] R. B. Mann and S. N. Solodukhin, Phys. Rev. D**54**, (1996) 3932.

[23] A. Strominger and C. Vafa, Phys. Lett. B **379**, (1996) 99.

[24] J. D. Brown and M. Henneaux, Commun. Math. Phys. **104**, 207 (1986).

[25] A. Strominger, J. High Energy Phys. **9802**, (1998) 009.

[26] S. Carlip, Phys. Rev. Lett. **82**, 2828 (1999).

[27] S. Carlip, Entropy from conformal field theory at Killing horizon, gr-qc/9906126 (1999).

[28] J. Lee and R. M. Wald, J. Math. Phys. **31**, 725 (1990).

[29] R. M. Wald, Phys. Rev. D **48**, R3427 (1993).

[30] V. Iyer and R. M. Wald, Phys. Rev D **50**, 846 (1994).

[31] V. Iyer and R. M. Wald, Phys. Rev D **52**, 4430 (1995).

[32] R. P. Kerr, Phys. Rev. Lett.**11**, 237(1963).

[33] E. T. Newman et al., J. Math. Phys. **6**, 918 (1965).

[34] B. Carter, Commun. Math. Phys. **10**, 280 (191968).